Math 103: College Algebra                                                       NAME:
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# Study Guide for the Final in College Algebra

## 1    Easy questions

**1)**(3 points) True or false
   a) Every real number has an opposite (additive inverse).
   b) Every real number has a reciprocal (multiplicative inverse)
   c) $a(b + c) = (b + c)a$
   d) Every rational number is also a real number.

**2)**(3 points) Which one of the following algebraic expressions is a polynomial? If it is a polynomial, give the degree and the number of terms.
   a) $3x^2 - 2x + 1$
   b) $4x^3 + \frac{7}{2}x - 18$
   c) $2x^2 + 3x + \frac{1}{x}$

**3)**(2 points) Subtract: $(2x^2 + 12xy - 9) - (-x^2 + x - y - 5xy + 6)$.

**4)**(2 points) Simplify
$$(3x^2 - x + 4) - (x^3 + x + 2)$$

**5)**(2 points) Multiply $(2x^2 + 4)(x - 3)$.

**6)**(2 points) Factor the following polynomial completely:
$$f(x) = x^4 - 16$$

**7)**(2 points) Factor the following polynomial:
$$f(x) = x^3 - 4x^2 + 3x$$

**8)**(2 points) Simplify
$$\frac{x + 2}{x - 1} - \frac{3 - x}{x - 1}$$

**9)**(3 points) Simplify
$$\left(\frac{x + 2}{x - 1}\right)\left(\frac{x - 1}{x - 2}\right)$$

**10)**(1 point) Write $b^{-2}$ without negative exponent.

**11)**(2 points) Solve the following equation:
$$4 - 3(2 - x) = 2 - (x - 1)$$

**12)**(3 points) Solve

$$
\begin{aligned}
2x + 6y &= -1 \\
4x - 3y &= 3
\end{aligned}
$$

**13)**(3 points) Solve the system of equations

$$
\begin{aligned}
2x - y &= 3 \\
x + 5y &= 2
\end{aligned}
$$

**14)**(2 points) Solve the inequality $3(x - 2) + 2(2 - 2(x + 2)) < 5x - 1$ and graph the solutions.

**15)**(2 points) Multiply the two complex numbers $1 + 3i$ and $2 - i$ and write the result in standard notation.

**16)**(2 points) Simplify $(4 + 3i) + (4 - 3i)$, where $i$ denotes the imaginary unit.

**17)**(3 points) Solve the equation $2x^2 = 3x + 1$.

**18)**(2 points) Solve the following equation:

$$
x^2 + 2x - 35 = 0
$$

**19)**(2 points) What is the distance between the points $(-3, 1)$ and $(1, 2)$?

**20)**(2 points) Find the center and the radius of the circle given by the equation

$$
(x + 1)^2 + (y - 2)^2 = 4.
$$

Then graph the circle.

**21)**(2 points) Find the equation for the circle with center $(2, -3)$ and radius 5.

**22)**(3 points) Draw the graph of the function $f(x) = -1 + 2x$. What is the slope? Find also both intercepts.

**23)**(3 points) Find the slope of the straight line through the two points $(-1, 1)$ and $(2, 3)$.

**24)**(3 points) Given $f(x) = x\sqrt{x - 1}$, find $f(0)$, $f(3)$, and $f(4x^2 + 1)$, and simplify the latter.

**25)**(3 points) Given are the functions $f(x) = x^2 + 3$ and $g(x) = \sqrt{x - 1}$. Find the functions $f + g$, $fg$, $f \circ g$ (simplify the expressions where possible), and find their domains.

**26)**(4 points) Which of the following four graphs are graphs of functions, which of them are graphs of 1-1 functions.

**27)**(2 points) Find all solutions of the equation $(x - 4)(2x + 3) = 0$

**28)** (2 points) Which degree-3 polynomial has zeros 4, -2, and -5?

**29)** (3 points) Divide $x^4 + 8x^3 - 34x^2 + 10x - 21$ by $x - 3$

**30)** (2 points) The radioactive isoptope $^{99m}$TC has a half-life of 6 hours. If we start with 14 milligrams, how much will be present after 9 hours?

**31)** (2 points)
    a) Translate $\log_{27}(3) = \frac{1}{3}$ into exponential form:
    b) Translate $81^{-1/4} = \frac{1}{3}$ into logarithmic form:

**32)** (2 points) Express $\log_b(54) - \log_b(6)$ as a single logarithm.

**33)** (2 points) Express $\log_{10}(y^x)$ as a product.

**35)** (1 point) What is the inverse of the function $y = \ln(x)$?

# 2    Less easy questions

**36)** (2 points) Factor by grouping

$$x^3 - 2x^2 - 9x + 18$$

**37)** (3 points) Factor the following polynomial completely:

$$g(x) = x^4 - x^3 + 4x^2 - 4x$$

**38)** (3 points) Simplify

$$\frac{x+2}{x^2-1} - \frac{x-2}{(x-1)^2}$$

**39)** (3 points) Simplify

$$\frac{c + \frac{8}{c^2}}{1 + \frac{2}{c}}$$

**40)** (3 points) Simplify (assume that all exponents are integers):

$$\left( \left( \frac{x^r}{y^t} \right)^3 \left( \frac{x^{3r}}{y^{2t}} \right)^{-2} \right)^{-2}$$

**41)** (3 points) Find all solutions of the following equation:

$$\frac{8x^2 - 31x}{(x-5)(x-2)} = \frac{3x}{x-5} + \frac{4x}{x-2}$$

Don't forget to check your solution(s)!

**42)**(3 points) Solve
$$\frac{3}{x+2} + \frac{2}{x-2} = \frac{4x-4}{x^2-4}$$
. Check your solution.

**43)**(3 points) Solve
$$x + \frac{4}{x} > 4.$$

**44)**(2 points) Solve $|x+6| \leq 8$.

**45)**(3 points) Solve $|3x-4| \leq 5$.

**46)**(2 points) Factor the polynomial
$$x^2 + 2x - 35$$

**47)**(2 points) The diagonal of a square is 1.341 cm longer than a side. Find the length of the side.

**48)**(3 points) Solve the equation
$$\sqrt{x-3} + x = 5$$
Don't forget to check your solution(s).

**49)**(3 points) Solve $x^4 - 5x^2 - 36 = 0$.

**50)**(3 points) Solve $(x+3)(x-2)(2x-3) < 0$.

**51)**(3 points) Solve $\frac{(x+3)(2x-3)}{x-4} < 0$.

**52)**(3 points) Is the following equation the equation of a circle? If it is, give the center and the radius.
$$x^2 + 3x + y^2 - 5y - \frac{1}{2} = 0$$

**53)**(3 points) Find both intercepts and the slope-intercept form of the straight line that goes through the point $(3, -2)$ and $(-4, 3)$.

**54)**(3 points) Put the quadratic equation $f(x) = x^2 - 4x + 2$ into the form $f(x) = a(x-h)^2 + k$ and find $x$- and $y$-coordinates of its vertex.

**55)**(3 points) Find the vertex of the quadratic function $f(x) = x^2 - 6x + 1$ and graph it. What are domain and range of it?

**56)**(2 points) Determine whether the functions are inverses of each other: $f(x) = \frac{2x-5}{4x+7}$ and $g(x) = \frac{7x-4}{5x+2}$.

**57)**(3 points) Find all solutions of the equation $(x^2 - 4)(2x+3) = 0$

**58)**(2 points) The function $f(x) = \frac{x-2}{x+1}$ is 1-1. Find its inverse function.

**59)**(3 points) The function $f(x) = \frac{3}{2-x} + 1$, shown in the graph below, is 1-1. Find the defining expression for the inverse function, and graph the inverse function into the same coordinate system.

**60)**(2 points) Use the rational zero theorem to find all **rational** zeros of the polynomial

$$x^3 - 9x^2 + 21x - 5.$$

**61)**(3 points) Sketch a graph of the function $f(x) = (x+3)(x-2)(x+1)$.

**62)**(1 point) Use your calculator to compute $\log_5(9)$.

**63)**(2 points) Solve the equation $e^{2x+1} = 4$.

**64)**(3 points) Solve $2 + \log_5(8 - 7x) = 5$.

**65)**(3 points) You invest 3000 Dollar, at a interest rate of 5 Percent, compounded quaterly. Find the length of time required for the investment to grow to 10,000 Dollar.

**66)**(4 points) The cost of a Hershey chocolate bar in 1962 was 5 cents and was increasing at an exponential growth rate of 9.7 %.

a) Find an exponential function describing the growth of the cost of a Hershey bar.

b) What will a Hershey bar cost in 2008?

c) When will a Hershey bar cost $ 15?

**67)**(4 points) The Van Gogh paining *Irises* sold for $ 84,000 in 1947, but was sold again for $ 53,900,000 in 1987. Assuming that the growth in the value $V$ of the painting was exponential:

a) Find the formula for for the value $V$, assuming $P_0 = 84,000$.

b) Estimate the value of the painting in 2007.

c) After what amount of time will the value of the painting be $ 1,000,000,000 ?

**68)**(3 points) a) If you invest $ 2000 in an account paying 5%, compounded monthly, how much money will be in the account at the end of 7 years? Use the formula $P(t) = P_0(1 + \frac{r}{n})^{nt}$.
 b) With the same data as in (a), how long do you have to wait until your money doubles?

**69)**(3 points) A car travels 140 mi at a certain speed. If the speed had been 20 mph faster, the trip could have been made in 3 hours less time. Find the speed.

# 3   More difficult questions

**70)**(4 points) Simplify
$$2 - \frac{1}{1 - \frac{2}{x+1}}$$
(represent as one simple fraction, reduced to lowest terms)

**71)**(4 points) Solve. Don't forget to check!
$$\frac{2}{x-2} = \frac{5}{x-3} - \frac{1}{x+1}$$

**72)**(4 points) Solve
$$|\frac{x+3}{x-2}| < 2.$$

**73)**(4 points) Find all solutions of the following equation:
$$\frac{8x^2 - 31x}{(x-5)(x-2)} = \frac{3x}{x-5} + \frac{4x}{x-2}$$
Don't forget to check your solution(s)!

**74)**(4 points) Use the substitution $u = \sqrt{\frac{x}{x-1}}$ to solve the equation
$$\frac{x}{x-1} - 6\sqrt{\frac{x}{x-1}} - 40 = 0.$$

**75)**(4 points) Solve $(1 + \sqrt{x})^2 + (1 + \sqrt{x}) - 6 = 0$.

**76)**(4 points) Solve $\sqrt{6x+7} - \sqrt{3x+3} = 1$. Don't forget to check!

**77)**(4 points) Solve the equation
$$\frac{x}{x^2-1}^2 - 2\frac{x}{x^2-1} - 15 = 0$$
.

**78)**(4 points) Solve the inequality $\frac{x^2-x-20}{x+1} \le 0$, and graph its solution set.

**79)**(4 points) Find the center and the radius of the circle given by the equation
$$x^2 + y^2 + 8x - 4y + 10 = 0$$
Then graph this circle. (Use the completing the square method).

**80)**(4 points) a) Find one zero of the polynomial function $P(x) = x^3 - 6x^2 + 5x + 12$, using the rational zero theorem and the fact that one rational zero lies between

b) Use long (or synthetic) division and the quadratic formula (or completing the square) to find **all** roots of the polynomial $P(x)$.

**81)**(4 points) a) Determine which one of the numbers 2 or 3 is a root (zero) of $P(x) = x^3 - 5x^2 + 10x - 8$.
   b) Find **all** roots of the polynomial.

**82)**(4 points) Beth can paint a room twice as fast as Adam. If both would work together, they would need 3 hours and 12 minutes. How long would Adam need when working alone?

**83)**(4 points) Liza alone could paint a room 2 hours quicker than Jim alone. If they would work together, they would need 5 hours. How long would Jim need alone?

**84)**(4 points) How many cl of a 84% solution should be added to 200 cl of a 49% solution to get a 70% solution?

**85)**(4 points) You want to create 11 liters acid solution of 80% concentration. How much of a 25% acid solution do you have to add to 8 liters of another acid solution to get the desired solution?

**86)**(4 points) A boat goes downstream for 6 hours. For the same distance back upstream it needs 11 hours The speed of the boat in still water is 10 mph. What is the speed of the stream?

**87)**(4 points) A boat goes 20 miles downstream. On the way back, upstream, it needs for the same distance 5 hours. What is the speed of the stream if the speed of the boat in still water is 8mph?

**88)**(4 points) A rectangular garden is 60 ft by 80 ft. Part of the garden is torn up to install a sidewalk of uniform width around the garden. The new area of the garden is 2/3 of the old area. How wide is the sidewalk?

**89)**(4 points) A 15 ft ladder leans against a wall. The bottom of the ladder is 4 ft from the wall. The bottom is then pulled out 3 ft farther. How much does the top end move down the wall.