

Midterm Exam

There are 3 pages with 10 questions.

This midterm is, as all our tests and quizzes, a closed-book-and-notes exam. Please note that calculators, even graphic calculators, even the Computers and Math-Cad are allowed. Please show all your work. GOOD LUCK!

1) (8 points) Differentiate

a) $f(x) = x^4 - 3x^3 + x^2 - x + 1$

b) $f(x) = \frac{3}{x^2}$

c) $f(x) = (x^2 + 2)(\sqrt{x} - 1)$

d) $f(x) = (x^3 + 5x - 2)^4$

e) $f(x) = \sqrt{\frac{3x+1}{2x-1}}$.

2) (2 points) For the function $f(x) = \frac{2}{x^2+2}$, find $f'(x)$ and $f''(1)$.

3) (4 points) A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. Express the length of the fence as a function of one side of this rectangular field. Draw this function, and by looking on the graph, decide which shape of the field would minimize the length of the fence.

4) (3 points) Draw the graph of a function f obeying the following requirements:

- $f(1) = 2$
- $f'(2) = -1$
- $f'(1) = 1$

5) (3 points) The folium of Descartes is given by the equation

$$x^3 + y^3 = 3xy$$

. Find the slope of the tangent line at the point $(3/2, 3/2)$ and also at the point $(4/3, 2/3)$.

8) (3 points) Indicate which of the following sentences is true respectively false:

- The graph of the derivative $f'(x)$ is the tangent line of f .
- The slope of the tangent line to f at $x = 2$ equals $f'(2)$
- A function f is differentiable at a if $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ exists.

9) 2 points) Consider a function $y = f(x)$ whose tangent line at $x = 2$ has the equation $y = 1 + \frac{x}{3}$. Find $f(2)$ and $f'(x)$ for the original function f . Is it also possible to tell something about $f(3)$?

10) (3 points) Find the equation of the tangent line to the curve of the function $f(x) = \frac{x+1}{x-1}$ at $x = 2$.