

Midterm Exam

There are 33 possible points in 10 questions.

This midterm is, as all our tests and quizzes, a closed-book-and-notes exam. Please note that calculators, even graphic calculators, are allowed, but notebooks, organizers, palm computers, cell phone, ... everything where you can store text, is **not allowed**.

Please show all your work. GOOD LUCK!

1) (6 points) Differentiate

a) $f(x) = (x + \frac{2}{x})^2$

b) $f(x) = (2x - \frac{1}{x})(\sqrt{x} + x)$

c) $f(x) = (x^3 - 3x + 7)\sqrt{x^4 + 2x^2 + 8}$.

2) (2 points) For the function $f(x) = \frac{x^2-4}{x-2}$, find $f'(x)$ and $f''(1)$.

3) (3 points) Find all critical points, and all the relative minimum or maximum points of the function

$$f(x) = \sqrt{x^2 + x + 1}.$$

Use first derivative test.

4) (3 points) Draw the graph of a function f obeying the following requirements:

- $f(1) = 2$
- $f'(2) = -1$
- f is concave down everywhere.

5) (2 points) What is a local minimum? At which points can a continuous function have a local minimum?

- 6) (4 points) Look at the four functions whose graphs are given below.
- a) Which of these functions have a positive first derivative for all x ?
 - b) Which of these functions have a negative second derivative for all x ?

- 7) (3 points) Find the slope of the curve given by the equation

$$x^2 + x^2y + y^3 = 9$$

at the point $(2, 1)$

8) (4 points) Indicate which of the following sentences is true respectively false:

- a) If a function is concave up at a , then $f'(a) > 0$.
- b) If a function has a local maximum at a , then $f'(a) = 0$.
- c) A function f is differentiable at a if $\lim_{h \rightarrow a} \frac{f(a+h)-f(a)}{h}$ exists.
- d) If a function is continuous at a point a , then it is differentiable at a .

9) 2 points) Consider a function $y = f(x)$ whose tangent line at $x = 2$ has the equation $y = 1 - \frac{x}{3}$. Find $f(2)$ and $f'(x)$ for the original function f .

10) (4 points) Find all relative extreme points (minima and maxima) of the function $f(x) = (x + 1)^{1/3}$. Where is the function increasing, where decreasing, where is it concave up, where is it concave down?