

Midterm Exam

There are 32 possible points in 8 questions.

This midterm is, as all our tests and quizzes, a closed-book-and-notes exam. Please note that MathCad and calculators, are allowed, but notebooks, organizers, palm computers, cell phone, ... everything where you can store text, is **not allowed**.

Please show all your work. GOOD LUCK!

1) (8 points) Differentiate (MATHCAD is banned for this one question).

a) $f(x) = 5x^4 - 4x^2 + 3x - 11$

b) $f(x) = \frac{1}{x^2} + 2\sqrt{x^3}$

c) $f(x) = (x^3 - 3x + 4)(x^3 - x + 4)$

d) $f(x) = \frac{1}{\sqrt{x^3 - 2x - 1}}$

e) $f(x) = \left(\frac{2\sqrt{2x+1}+x}{2x+1}\right)^5$

2) (3 points) Find the equation of the straight line that is tangent to the graph of the function $f(x) = (2x^2 + 1)^3$ at $x = -2$.

3) (5 points) A rocket is shot vertically into air. Its height x seconds after the start can be expressed by the function $h(x) = 90x - 3x^2$. The temperature y at height u follows the equation $y(u) = \sqrt{200 - \sqrt{15}u}$ (in degrees celsius) on that summer day.

- a) What is the height of the rocket 10 seconds after start?
- b) What is the instantaneous speed of the rocket after 10 seconds?
- c) What is the instantaneous acceleration of the rocket after 10 seconds?
- d) What is the temperature at the rocket 10 seconds after start?
- e) What is the instantaneous rate of change of temperature at the rocket 10 seconds after start?

4) (1 point)

a) How is the derivative f' of a function defined?

5) (4 points) Indicate which of the following sentences is true respectively false:

a) The derivative of a function is its tangent line.

b) If a function is continuous at a point a , then it is differentiable at a .

c) A function f is differentiable at a if $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ exists.

d) Every function has a global (=absolute) maximum.

6) (4 points) Find local minima and maxima of the function $f(x) = x\sqrt{x+3}$. Find also for what x the function is increasing and decreasing.

7) (3 points) Find all absolute (=global) minimum and maximum values of the function $f(x) = \frac{x}{x^2+1}$, defined on the closed interval $0 \leq x \leq 2$.

Next, please answer **only one** of the following two questions: (if you do both, one serves as bonus question)

8a) (4 points) A rectangle is inscribed in a right triangle, as shown in the figure below. If the triangle has sides of length 5, 12, and 13, express the area of the inscribed rectangle as a function of its width. Graph the function. For which width do we get the largest area?

8b) (4 points) A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point (4,2), its x -coordinate increases at a rate of 3 units/sec. How fast is the distance from the particle to the origin changing at that instant?