

Midterm Exam

There are 29 possible points in 8 questions.

This midterm is, as all our tests and quizzes, a closed-book-and-notes exam. Please note that MathCad and calculators, are allowed, but notebooks, organizers, palm computers, cell phone, ... everything where you can store text, is **not allowed**.

Please show all your work. GOOD LUCK!

1) (8 points) Differentiate (MATHCAD is banned for this one question).

a) $f(x) = 3x^4 - 3x^2 + 2x - 1$

b) $f(x) = \frac{1}{x^4} + 3\sqrt{x}$

c) $f(x) = (x^4 - 6x + 7)(x^3 - 2x + 2)$

d) $f(x) = \frac{x^3+x^2+x+1}{\sqrt{x^3-x-1}}$

e) $f(x) = \sqrt{(x^3 + x^2 + x + 1)(x^3 - 2x - 1)}$

2) (3 points) For the function $f(x) = \frac{x+2}{x-3}$, find $f'(x)$, $f'(2)$, and $f''(2)$.

3) (3 points) Below, the graph of the function f is given. Estimate $f(1)$, $f'(1)$, and $f'(0)$. Also tell for which values of x the derivative f' is negative.

4) (4 points) Below the graph of the equation

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$$

is displayed. Draw the tangent for the curve at the point $(0, 0.5)$, and use implicit differentiation to find the slope of that tangent line. Find also all horizontal and all vertical tangents to the curve.

5) (3 points)

- a) For a function f , what exactly does $f'(3)$ represent? Is it a number, an expression, a function, or an equation?
- b) Is there a function f , where $f(100) - f(99)$ and $f'(100)$ are equal? Which one? Is there a function where these two values differ by at least 3?

6) (4 points) A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. A ladder starts outside, leans on the fence, and goes just to the wall. Express the height of the highest position of the ladder (where it leans against the wall) in terms of the length x of the ladder. Graph the function as well.

7) (3 points) Find the equation of the straight line that is tangent to the graph of the function $f(x) = \sqrt{5x^2 + 1}$ at $x = -1$.