Math 200—Calculus
Fall 2006
Franklin College
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NAME:

Study Guide for Calculus — MAT200

1 Part I—MathCad is not allowed here

1) Differentiate
a) $f(x) = x^3 + 4x^2 + 2x + 4$,
b) $f(x) = \ln(3x) - \sqrt{x}$,
c) $f(x) = \frac{1}{x} + e^{3x-1}$
d) $f(x) = (x^2 + x + 1)\ln(x)$,
e) $f(x) = e^{3+ln(x)}$,
f) $f(x) = \frac{3x-1}{2x+1}$,
g) $f(x) = \frac{x-1}{x+1}$
h) $f(x) = \sqrt{x^3 - x + 2}$,
i) $f(x) = e^{3+ln(x)}$,
j) $f(x) = \ln(x+1)\ln 2x + 1$,
k) $f(x) = \ln(\sqrt{x^3 + 5x + 1})$,

2) Find antiderivatives of
a) $f(x) = x^4 - 2x^3 + 5$,
b) $f(x) = \sqrt{x} + 2e^x$,
c) $f(x) = \frac{1}{x^4}$,
d) $f(x) = 2\sqrt{x} + \frac{1}{x^2}$,
e) $f(x) = \ln(3) + e^{2x} + x$,
f) $f(x) = xe^x$,
g) $f(x) = (x+1)\sqrt{x^2 + 2x + 5}$,
h) $f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$, (use substitution with $u = e^x + e^{-x}$)
i) $f(x) = x\ln(x^{2/3})$, (use integration by parts)
j) $f(x) = \frac{\ln(x)}{x^2}$,
k) $f(x) = \frac{x}{x+1}$.
l) $f(x) = x\ln(x^{2/3})$, (use integration by parts)

2 Part II—MathCad is allowed

3) (3 points) Find whether each of these limits exists. If it does, find its value.
a) $\lim_{x\to 2} \frac{x-2}{x-1}$ b) $\lim_{x\to 2} \frac{x-2}{x^2-4}$

4) (2 points) For the function $f(x) = \frac{x^2-4}{x+2}$, find $f'(x)$ and $f''(3)$.

5) Given is the function

$$f(x) = (x^2 - 3x + 1)(\sqrt{x} + 1).$$

Find the equation of the tangent line to the graph of f at $x_0 = 1$.

6) The following graph describes the height y of a rocket depending on the time t in seconds during the first four seconds. What is the average speed of the rocket during the first three seconds? What is the instantaneous speed of the rocket after 2 second?

7) (3 points) Find the slope of the curve given by the equation

$$x^2 + x^2y + y^3 = 9$$

at the point $(2, 1)$

8) (4 points) Below the graph of the equation

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$$

is displayed. Draw the tangent for the curve at the point $(0, 0.5)$, and use implicit differentiation to find the slope of that tangent line. Find also all horizontal and all verticeal tangents to the curve.

9) (4 points) A plane, flying horizontally at an altitude of 1 mile and a speed of 500 miles/hour passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles (measured on the ground) away from the station. (Note: Of course, Pythagoras' Theorem must be applied again.)

10) (3 points) Find all critical points, and all the relative minimum or maximum points of the function

$$f(x) = \sqrt{x^2 - x + 1}.$$

Use first derivative test.

11) (4 points) Determine the intervals where the function

$$f(x) = \ln(2x - 2) - x^2$$

is increasing, decreasing, concave up and concave down. (Hint: To check your work, graph the function and have a good look at it.)

12) (4 points) Determine where the function

$$f(x) = (x^2 - x)^2$$

is increasing, decreasing, concave up and concave down.

13) (4 points) Find all relative extreme points (minima and maxima) of the function $f(x) = (x-1)^{1/3}$. Where is the function increasing, where decreasing, where is it concave up, where is it concave down?

22) (4 points) An open box with a square base should be constructed. The material for the bottom of the box costs 5 Sfr per square meter, and the material for the sides costs 2 Sfr per square meter. We want to create a box with maximum possible volume, given that we have 400 Sfr to spend. What are the dimensions of the box?

23) (4 points) A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. A ladder starts outside, leans on the fence, and goes just to the wall. Express the height of the highest position of the ladder (where it leans against the wall) in terms of the length x of the ladder. Graph the function as well.

24) (3 points) A cylindrical can with no top has beem made from 8π square inches of metal. Express the volume of the can as a function of its radius. (Note that the volume of the can equals area of the bottom times height.) (3 points)

25) (4 points) Given is the graph of a function f below. Roughly sketch its tangent at $x = 2$, the first and second derivative, f' and f'', and some antiderivative F.

26) (3 points) Draw the graph of a function f obeying the following requirements:

- $f(1) = 3$
- $f'(2) = -1$
- f is concave up everywhere.

27) (4 points) Look at the four functions whose graphs are given below.
a) Which of these functions have a negative first derivative for all x?
b) Which of these functions have a positive second derivative for all x?

28) 2 points) Consider a function $y = f(x)$ whose tangent line at $x = 2$ has the equation $y = 1 - \frac{x}{2}$. Find $f(2)$ and $f'(x)$ for the original function f.

39) (6 points) A rocket is shot vertically into air. Its height x seconds after the start can be expressed by the function $h(x) = 90x - 3x^2$. The temperature y at height u follows the equation $y(u) = \sqrt{200 - \sqrt{15u}}$ (in degrees celsius) on that summer day.
a) What is the height of the rocket 10 seconds after start?
b) What is the average speed of the rocket during the first 10 seconds?
c) What is the instantaneous speed of the rocket after 10 seconds?
d) What is the instantaneous acceleration of the rocket after 10 seconds?
e) What is the temperature at the rocket 10 seconds after start"?
f) What is the instantaneous rate of change of temperature at the rocket 10 seconds after start?

40) (4 points) Which of the statements below is true, which one is false. If it is false, briefly explain why it is false. If you think there is a typo in any of the formulas, correct it.

a) If a function is concave up at a, then $f''(a) > 0$.

b) If $f'(a) = 0$, the f has a local maximum at a.

c) A function f is differentiable at a if $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ exists.

d) If a function is continuous at a point a, then it is differentiable at a.

e) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$,